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THE DEVELOPMENT OF THE THIN-FILM NAPHTHALENE MASS-TRANSFER ANALOGUE TECHNIQUE FOR THE DIRECT MEASUREMENT OF HEAT-TRANSFER COEFFICIENTS

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Abstract—The thin-film naphthalene mass-transfer analogue technique has provided a rapid and economic method of comparing forced convection heat-transfer coefficients in a variety of situations. The present contribution outlines a further development of the technique to the stage that it is capable of predicting absolute heat-transfer coefficients directly and accurately. Heat-transfer coefficients derived by analogy for flow through a tube agree with those given by the Colburn equation to within ± 5 per cent over a Reynolds number range $7{\cdot}6 \times 10^4 < Re < 1{\cdot}3 \times 10^6$. Tube entrance region heat-transfer measured by analogy is in good agreement with other investigations.

NOMENCLATURE

- A, surface area of perspex plug;
- c, specific heat capacity of air at constant pressure;
- C_w, wall concentration of naphthalene vapour;
- C_∞, free stream concentration of naphthalene vapour;
- c_f, local surface friction factor;
- h, heat-transfer coefficient;
- h_m, mass-transfer coefficient;
- p_w, saturated vapour pressure of naphthalene;
- q, heat flux;
- R_n, gas constant for naphthalene;
- T, absolute temperature;
- u, bulk mean air velocity;
- Nu, Nusselt number;
- Pr, Prandtl number;
- Pr_t, turbulent Prandtl number;
- Re, Reynolds number;
- Sc, Schmidt number;
- Sc_t, turbulent Schmidt number;
- St, Stanton number;
- St_m, mass Stanton number.

Greek symbols

- ρ, air density;
- ϕ, heat/mass transfer analogy factor ($= St/St_m$).

INTRODUCTION

MEASUREMENT of the sublimation rate of solid naphthalene is a well proven method for determining mass-transfer coefficients in convective situations. Furthermore, the analogous form of the equations for heat and mass transfer has led numerous workers, e.g. [1–3], to employ the technique as a convenient method of predicting heat-transfer coefficients. The accuracy of the method is dependent, however, upon precise measurements of small profile changes, and upon the form of the analogy employed. Goldstein *et al.* [4] have used the method to measure natural convection adjacent to horizontal plane surfaces, and these workers employed an accurate weighing technique to determine naphthalene sublimation rates. Wilkie and White [5] used a modified form of the method for examining the variation of heat-transfer coefficient along the ribbed surfaces of AGR nuclear fuel elements. The fuel element was coated with a very thin film of naphthalene and mounted in a perspex channel through which air was drawn. Transfer coefficients over the surfaces were compared by observing the times taken for the naphthalene to clear from each point. This thin-film technique provided a particularly simple and economic method of comparing transfer coefficients, and has since been used in a number of alternative applications, for example, in power station boiler headers [6]. In all these applications, the thin-film technique has been used only on a comparative basis, and this has, of course, limited its usefulness to a considerable extent. The development of the technique to measure absolute heat-transfer coefficients directly would obviously broaden its scope appreciably. This has been the object of the work described in this paper.

The Chilton–Colburn analogy [7] has been used extensively for predicting heat-transfer coefficients from mass-transfer experiments, but Lewis [8] has pointed out the limitations of this simple analogy with its constant empirical factor $(Sc/Pr)^{2/3}$ to account for the difference in heat and mass diffusivities. A much more rigorous treatment of the analogy is needed if accurate heat-transfer coefficients are to be obtained. The present contribution discusses the form of the analogy, outlines the development of the thin-film technique for the absolute measurement of heat-transfer coefficients, and describes a series of tests on a long straight tube to compare heat-transfer coefficients measured by analogy with those from other sources.

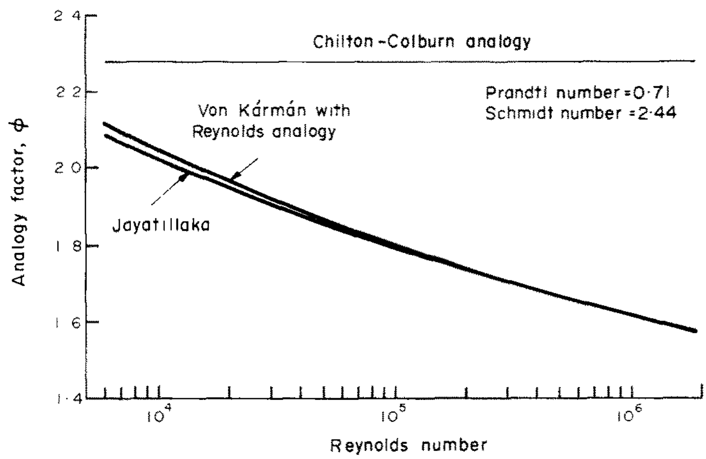

FIG. 1. Graphs of the heat/mass transfer analogy factor ϕ $(= St/St_m)$ against Reynolds number for fully developed flow in a tube.

THE FORM OF THE ANALOGY

A rigorous form of the heat/mass transfer analogy must take account of the varying contributions of molecular and turbulent diffusivity across the boundary layer. Well established values for molecular diffusivities exist, but turbulent diffusivities are difficult to measure. Such measurements generally show considerable scatter, see for example Simpson and Field [9], or Blom [10] but investigators seem to agree that the ratio of the turbulent diffusivities of mass and heat (or turbulent Lewis number) is close to unity.

The modified von Kármán [11] universal velocity profile concept may be employed to derive an expression for the temperature difference across a turbulent boundary layer in terms of friction factor and Prandtl number. This concept does, of course, make the assumptions that (a) the turbulent diffusivities of heat and momentum are equal (i.e. turbulent Prandtl number $Pr_t = 1$), (b) the molecular and turbulent diffusivities are additive in the buffer region and (c) the turbulent diffusivities are zero in the sub-layer. The equivalent expression for Stanton number using this concept is

$$St = \frac{c_f/2}{1+5\left(\dfrac{c_f}{2}\right)^{1/2}\left[Pr-1+\ln(\tfrac{5}{6}Pr+\tfrac{1}{6})\right]}.$$

The corresponding expression for mass Stanton number St_m is obtained merely by substituting Schmidt number for Prandtl number in the above equation.

The values of the analogy factor $\phi(= St/St_m)$ derived from this analysis are shown in Fig. 1 for fully developed flow in a tube, where they are compared with the constant Chilton–Colburn analogy value of 2·28. These curves are derived from values of Prandtl number appropriate to air and of Schmidt number appropriate to naphthalene. It can be seen that the discrepancy between the two analogies increases with Reynolds number. The Chilton–Colburn value is some 40 per cent high at $Re = 10^6$.

More recently, Jayatillaka [12] has given an extensive review of the various proposed distributions of turbulent:molecular momentum diffusivities across the boundary layer, and suggested an empirical expression to describe heat transfer in the near-wall region. The expression for Stanton number derived from his recommendation is

$$St = \frac{(c_f/2)^{1/2}}{Pr_t\left(\dfrac{2}{c_f}\right)^{1/2} + A\left[\left(\dfrac{Pr}{Pr_t}\right)^{3/4}-1\right]\left[1+0{\cdot}28\exp\left(-0{\cdot}007\dfrac{Pr}{Pr_t}\right)\right]}$$

with $A = 8{\cdot}32$ for $Pr_t = 0{\cdot}9$ and $A = 9{\cdot}00$ for $Pr_t = 1$.

Again, values for ϕ are derived by substituting the appropriate values of Sc and Sc_t for Pr and Pr_t in the above formula to calculate St_m. Current opinion favours a value of 0·9 for Pr_t and Sc_t for flow in a pipe, and this value has been adopted in the present investigation. The variation of ϕ with Reynolds number derived from Jayatillaka's formula is included in Fig. 1, and it is this derivation that has been used in the current tests. It will be seen that there is in fact not much difference between the Jayatillaka values and those derived from the von Kármán analysis.

Although the above expression, when used in conjunction with the corresponding expression for mass Stanton number, represents a rigorous form of the heat/mass transfer analogy, knowledge of the friction factor is still required. This does, therefore, suggest that its use may be limited to geometries such as flat plates, tubes and annuli. Lewis [13], however, has used the technique very successfully to determine the heat-transfer variation in several other situations, including a finned tube in cross flow, with its associated regions of separated flow. The general principles that he has utilised to determine the variation of analogy factor over the finned surfaces could be applied to other geometries.

Finally, in this section, it should be remarked that for strictly analogous heat/mass transfer mechanisms, Prandtl and Schmidt numbers should be equal. The naphthalene mass-transfer analogue complies approximately in this respect for gases and many liquids. The analogy is tested much more severely when Prandtl and Schmidt numbers differ appreciably. The electrochemical mass-transfer analogue technique [14], for example, operates with Schmidt numbers typically around 1500 and above, and so considerable caution should be exercised in interpreting the results.

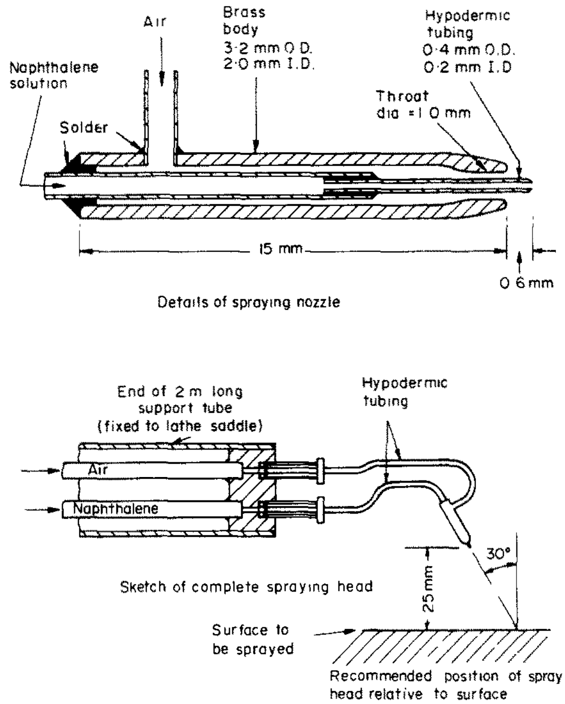

FIG. 2. Diagram showing the construction of the naphthalene spraying head.

REFINEMENTS TO THE SPRAYING TECHNIQUE

A critical stage in testing any component by the thin-film analogue method is the application of the naphthalene to the surfaces of interest. It is essential that a smooth coating of accurately uniform thickness is achieved, and considerable time has been spent in developing a reliable spraying technique. Some details of the spraying technique have been published previously [15], but since that time, further refinements have been made, both in the light of experience and to meet more exacting requirements, and these are now described for the benefit of any intending user.

The original spraying head has been improved by adopting a coaxial jet arrangement (Fig. 2), where naphthalene solution issuing from the centre nozzle mixes immediately with an annular jet of air, forming a finely atomized conical spray. The annular air jet sweeping the outside surfaces of the centre nozzle prevents any risk of build-up of solid naphthalene at the nozzle exit, which would otherwise result in a rapid blockage. This modified spraying head is extremely compact as the dimensions indicate. In most other respects the spraying facility is substantially as described in [15], except that very recently an automatic linear motion spraying table has been developed which enables flat surfaces to be sprayed uniformly. Cylindrical surfaces are sprayed very simply, of course, by mounting them in a lathe with the spraying head attached to the lathe saddle. The naphthalene solution for the current investigation is made from crystalline naphthalene (P grade) from BDH Chemicals Ltd, Poole, at a 20 per cent concentration in "Inhibisol", a non-toxic solvent manufactured by The Penetone Co. Ltd., Cramlington, Northumberland. A suitable naphthalene film thickness is approximately 0·015 mm, obtained typically with four passes at a solution flow rate of 60 mm^3/s and a linear surface speed of 0·8 m/s.

Further refinement of the spraying technique became necessary to meet the added requirement that the exact loading of naphthalene over the surfaces needed to be known before the analogy could be employed absolutely. Detailed investigations revealed that for a number of reasons, metering of the naphthalene solution by Rotameter alone was too unreliable and that direct weighing must be employed. This can be achieved by incorporating a flush-fitting removable plug in the surface to be sprayed. During the course of these investigations, it became apparent that the surface finish of the naphthalene coating was influenced by such factors as the distance between spraying nozzle and surface, the angle at which the spraying jet struck the surface, and the pressure of the coaxial atomizing air jet. A simple series of supplementary tests was conducted to determine the appropriate spraying distance and angle to give the best surface finish (see Fig. 2). Finest atomization was achieved using a relatively low airline pressure, of order 0·4 bar gauge pressure, which gave a very smooth surface finish. It is important to ensure that the surface finish is smooth, otherwise some particles may be physically blown off, and furthermore, a roughness effect could be introduced, both of which would tend to increase the mass-transfer rates.

The magnitude of any "blown particle" effect was examined over a range of air velocities in a series of tests on a sprayed tube with a perspex plug incorporated, in which each test was interrupted a number of times for the plug to be weighed. Any solid particles of naphthalene blown off at the start of the test would have shown up as an increased weight loss over this period. Figure 3 shows a typical graph of weight loss with time, which indicates an absolutely uniform rate of sublimation over all but the last stages of clearance. There was, therefore, no evidence of solid particle removal, a situation which held over the entire range of air velocity. These tests did, however, reveal the fall in sublimation rate during the final stages of clearance, which led to a false indication of the sublimation rate as illustrated in Fig. 3. For the particular naphthalene loading used for these tests, the error

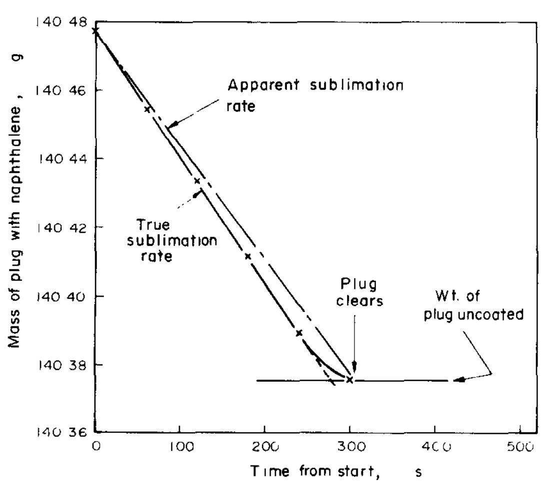

FIG. 3. The reduction in weight of a naphthalene coated plug set in the wall of a similarly coated perspex tube through which air is flowing.

introduced by this effect was of order 10 per cent, this factor being virtually independent of Reynolds number. The error would obviously increase with reduction in the initial naphthalene loading, and a simple correction was subsequently built into the calculation procedure to make allowance for the effect at any loading.

TEST SECTION AND PROCEDURE

The accuracy of the naphthalene thin-film mass/heat transfer analogy was determined in a series of tests on a straight tube over a range of air flows. The tube was fitted with a bellmouth intake that had been carefully machined from laminated wood and varnished to a very smooth finish, in order to give a uniform velocity profile at entry to the test section. A boundary-layer trip, in the form of a 3 mm wide strip of masking tape, was fitted in the throat of the intake, to ensure that the boundary layer was turbulent from the start of the test section. The intake was located in the wall of a large plenum chamber, into which air was drawn from the atmosphere through a plate-fin type of heat exchanger designed to provide a stable air temperature through the test section. Water was circulated through the heat exchanger from a temperature controlled reservoir, set at ambient temperature. Earlier tests without the temperature control showed inlet temperature fluctuations of up to 4°C during high velocity runs, due to temperature stratification effects within the laboratory. Such fluctuations are obviously undesirable since the sublimation process is very sensitive to temperature, owing to the rapid change in the saturated vapour pressure of naphthalene with temperature. A rise in the temperature of the naphthalene of 1°C increases the sublimation rate by 10 per cent. It was conceivable that the pressure–temperature relationship for naphthalene sublimation might be affected after being in solution with "Inhibisol". However, an i.r. analysis of a sample of sprayed naphthalene revealed no sign of any impurities, and so it was assumed that the relationship was unaffected.

The downstream end of the test section was attached through a smooth transition piece to ducting which connected to the suction side of two centrifugal fans. Two valves were incorporated in the ducting, one with fine adjustment to control the air mass flow and the other to provide a rapid on/off control. The air mass flow was metered by an accurately calibrated pitot-static tube located along the duct axis.

Two 0·5 mm dia, mineral insulated, sheathed nickel–chromium/nickel–aluminium thermocouples located in the plenum chamber were used to indicate the test section air entry temperature. A previously calibrated mercury-in-glass thermometer installed in a pocket on the outer wall of the test section served to check that the wall temperature remained constant to within 0·2°C during the duration of each test. A digital seconds counter recorded elapsed time from the start of a test.

The test section itself was a smooth, polished perspex tube 1·8 m long (the maximum length available commercially) and 0·127 m dia, incorporating a 100-mm dia removable plug for weighing purposes. The tube was coated with naphthalene over its entire length, representing a treated length of some fourteen diameters. Whilst this would not normally be considered a sufficient length in which to establish fully developed flow conditions, the evidence from many other workers indicates that heat-transfer coefficients would virtually have reached their fully developed value in this distance. Barbin and Jones [16], for example, in some very carefully executed tests, found that although fully developed flow was not attained within forty diameters of a bellmouth entry with boundary-layer trip, the wall shear stress attained its fully developed value within the first fifteen diameters. It is reasonable to assume that heat-transfer coefficients would behave in a similar manner to the wall shear stress (momentum transfer) in this respect. Mills [17], in a series of tests on a heated pipe with different entry sections, found that with the bellmouth and boundary-layer trip configuration, the Nusselt number reached a steady value between fifteen and twenty diameters downstream. In his investigations Mills did in fact use a uniform heat flux condition. No satisfactory reference to similar work with a uniform wall temperature condition, to which the naphthalene sublimation process is analogous, has been found in the literature, but two theoretical analyses [18, 19] which compared the two wall boundary conditions, both showed that the thermal entrance effect was shorter with a uniform wall temperature. In the light of these findings, it was concluded that a length of fourteen diameters was sufficient to ensure a fully established transfer condition for the purpose of the current tests.

The test procedure was very simple, although care was required at each stage in order to achieve accurate and repeatable results. The perspex section was cleaned thoroughly before spraying to remove any particles of dirt or grease drawn in from the atmosphere during the previous test. The removable plug was located accurately in the test section and clamped in position for spraying. During the spraying operation it was found that the plug became electrostatically charged, and it was necessary to remove this charge before weighing by earthing the plug, otherwise the charge upset the delicate balance mechanism to give an erroneous weight indication. The plug was weighed to the nearest 0·0001 g, after which it was replaced in position, and the test section mounted between the intake and transition section. The digital seconds counter was started as the on/off control was opened, and measurements made of air temperature, test section wall temperature, static pressure and dynamic head, and of the progress of clearance of naphthalene along the test section, at recorded intervals of time. When the test was completed, the plug was weighed once more so that the exact loading of naphthalene could be calculated.

TEST RESULTS

Tests were carried out over the Reynolds number range $7{\cdot}6 \times 10^4 < Re < 1{\cdot}3 \times 10^6$, and the results are presented in Figs. 4 and 5. Figure 4 shows the analogue values of heat-transfer coefficient for the downstream end of the tube compared with the Colburn equation values (20), and Fig. 5 compares the entrance region heat-transfer coefficient variation predicted by the analogue with results from several other investigations. The mass-transfer analogue calculation procedure is set out in the Appendix. It will be seen from Fig. 4 that

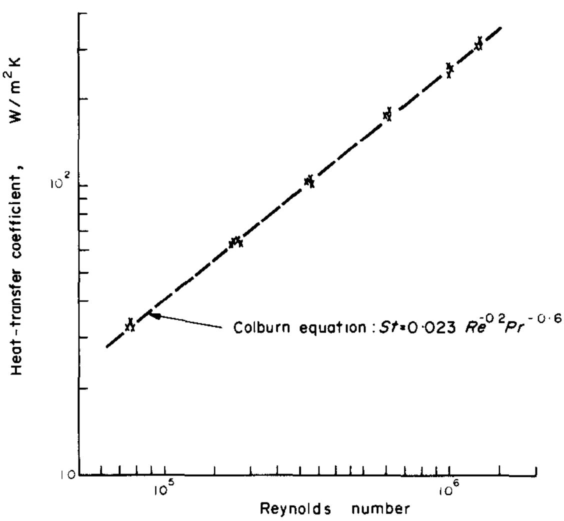

FIG. 4. Heat-transfer coefficients for fully developed conditions in a tube; mass-transfer analogue values compared with the Colburn equation.

the values of heat-transfer coefficient predicted by the analogue compare extremely well with those obtained from the Colburn equation, agreement being to within ± 5 per cent. It should be mentioned that the accuracy of the Colburn equation has been open to some doubt. Whilst much experimental evidence exists to confirm its accuracy, some recent evidence suggests that it may be a few per cent in error, e.g. [21]. It is outside the scope of the present paper to add to a familiar discussion. The Colburn equation is used for the present comparison on the basis that it is probably still the most widely recognised relationship giving a very good approximation to the available experimental data.

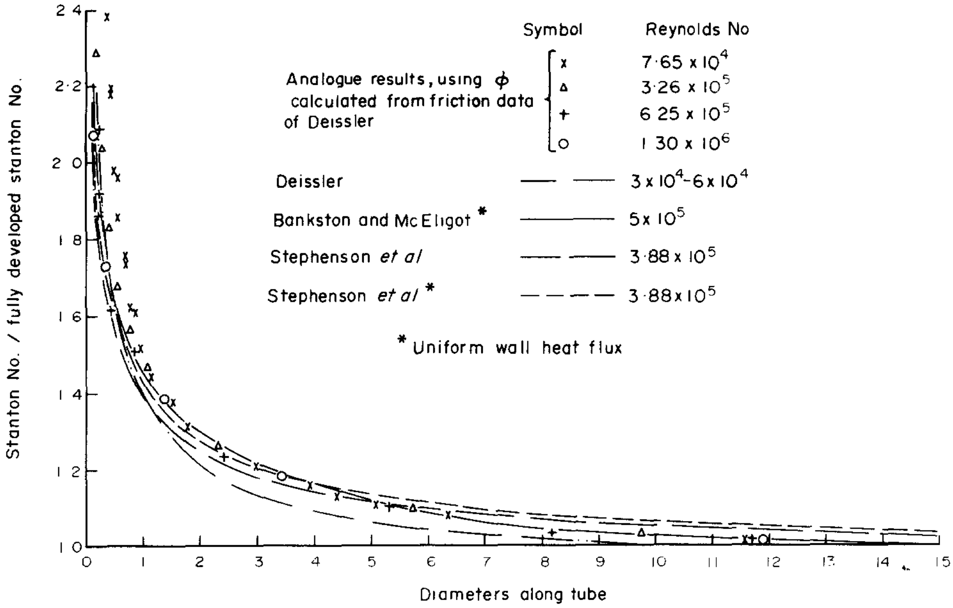

FIG. 5. Comparisons of tube entrance region heat-transfer variation.

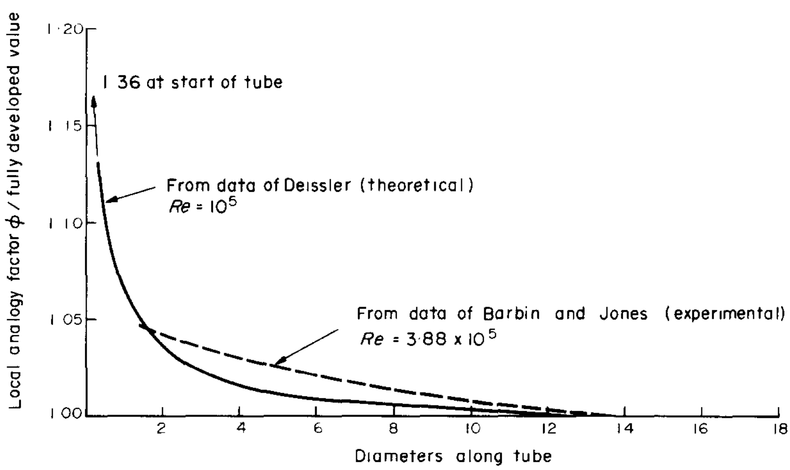

FIG. 6. The variation of the analogy factor ϕ for developing flow in a tube.

In calculating the tube entrance region heat-transfer variation from the mass-transfer analogue measurements, it had to be borne in mind that in the developing region of the tube, the value of the analogy factor ϕ is not constant, since the friction factor is changing along the tube. Barbin and Jones [16] have measured the shear stress variation in developing flow in a tube, and Deissler [22] has estimated the variation theoretically. These workers' results have been used to calculate the variation of ϕ in the entrance region of the tube, using Jayatillaka's formula [12], and Fig. 6 shows the values of ϕ along the tube expressed as a ratio of the fully developed value. In calculating the heat-transfer coefficient variations of Fig. 5, the same values of the ϕ ratio shown on Fig. 6 have been used for all Reynolds numbers. It is unlikely that this ratio changes significantly over the range of Reynolds numbers tested. In order to avoid confusion, the analogue results based on the friction factor data of Barbin and Jones have been omitted from Fig. 5 since the difference between these and those based on Deissler's friction data lies within the limits of experimental error.

As mentioned previously, the naphthalene sublimation process is analogous to a constant temperature wall condition in heat transfer, for which no satisfactory tube entrance region measurements have been published for purposes of comparison. Deissler [22] has investigated this situation analytically, and Fig. 5 compares the analogue results with his prediction. Deissler's prediction suggests that heat-transfer coefficients reach a steady value somewhat sooner than the analogue results indicate, and it is interesting to note that Mills [17] found the same situation when comparing his results with Deissler's prediction for the uniform wall heat flux condition. This suggests that Deissler's prediction is a little suspect in this respect.

More recently, Bankston and McEligot [23] have employed a finite difference computational procedure for predicting heat transfer in the entrance region of circular ducts, using a modified mixing length turbulence model. Unfortunately, these workers examined only the constant wall flux boundary condition, so that strictly speaking their results are not comparable with the analogue. However, Hall and Khan's [24] results for normally developing flow in a tube, i.e. without a boundary-layer trip, have shown that there is little difference between the two thermal boundary conditions for Reynolds numbers above 5×10^4. On the assumption that the same result is applicable to a tripped boundary layer, Bankston and McEligot's predictions may be compared with the present results. Included in Fig. 5 is their prediction for entrance region heat transfer at a Reynolds number of 5×10^5, and it will be seen that agreement with the analogue results is virtually perfect. Furthermore, the small Reynolds number effect evident in the analogue results over the first two diameters is also predicted by Bankston and McEligot.

Very recently at the Central Electricity Research Laboratories, Stephenson *et al.* [25] have been comparing the effect of using various turbulence models to predict entrance region heat transfer, again employing a finite difference procedure based on the method of Patankar and Spalding [26]. Included in their predictions is a comparison of the two wall boundary conditions, constant temperature and constant heat flux, for which they used a further modified form of mixing length turbulence model. This comparison is included in Fig. 5, for a Reynolds number of $3{\cdot}88 \times 10^5$, and confirms that at high Reynolds number, there is little difference in entrance region heat transfer between the two boundary conditions. Again, the predictions are in good agreement with the analogue results, although the predictions of Stephenson *et al.* do indicate a rather longer development length, of some twenty diameters. It is outside the scope of this paper to discuss these comparisons in detail, and this will be the subject of a future publication by Stephenson, Massey and Oliver. Suffice it to say here that the good agreement provides further evidence to confirm the accuracy of the thin-film analogue technique. In addition to achieving this principal objective, the present series of tests has revealed two other points worthy of mention, and these are discussed in the following sections.

BOUNDARY LAYER TRIPS

The method of boundary-layer tripping is well known as a means of promoting turbulent boundary-layer conditions from the very beginning of a developing flow. The selection of a suitable height of trip has, however, always been rather arbitrary, since no quantitative assessment of the performance of such trips has been undertaken, as this would be a very lengthy procedure. The thin film naphthalene technique is unique in that it demonstrates simply and vividly the effectiveness of the boundary-layer trip. In the present tests, a single layer of "Scotch" masking tape, 3 mm wide, was used for the first test runs at the highest flows, and its effectiveness was made immediately apparent by the pattern of clearance, which obviously corresponded to the condition of a fully turbulent boundary layer from the start. At lower flows, however, the situation was reached where the pattern of clearance changed. The clearance again started from the tube entry, but as time progressed, a second region of clearance appeared approximately two to three diameters downstream, spreading both upstream to meet the initial cleared portion, and downstream. This pattern of clearance indicated high coefficients at tube entry as before, a rapid decrease to a minimum about one and a half diameters downstream, followed by an increase to a secondary maximum and then the steady decline in coefficients towards the constant fully developed value; this variation is that obtained with a normally developing boundary layer. Thus, the single layer of masking tape was obviously no longer of sufficient height to trip the boundary layer, and progressively more layers were required as the flow was decreased. The pattern of clearance was in fact employed to determine the required boundary-layer trip thickness at each Reynolds number. At the lowest Reynolds number tested, seven thicknesses were necessary to achieve the desired fully turbulent boundary layer. During several tests, a critical situation was obtained where the trip was adequate in some positions round the tube, but not in others, giving a marked circumferential variation in the pattern of clearance near the start of the tube. The small variations in the height of the trip causing this situation were inevitable by virtue of its simple construction. The thin-film technique would obviously be very suitable for making a critical quantitative assessment of boundary-layer trip performance.

MODIFICATION TO BOUNDARY CONDITION

Wilkie and White [5] were first to point out that the mass-transfer boundary condition, equivalent to one of constant temperature, is modified by the appearance of bare patches during the thin-film sublimation process. The effect of these bare patches is to cause increased mass transfer from regions immediately downstream. An attempt was made during the present tests to estimate the magnitude of this "bare patch effect", but the complications of a developing hydrodynamic boundary layer, and of a falling away of the sublimation rate as clearance approached, introduced too many uncertainties. However, the evidence from the current work, which provided a very severe test of the effect, suggests that the observed transfer coefficients are not appreciably in error. In the entrance region of a tube, transfer coefficients fall away rapidly in the first one or two diameters, and in these circumstances, the bare patch effect would exert maximum influence. However, any significant advancement in clearance time in this region would reflect in a premature clearance at all positions downstream of at least the same order. Thus, the measured fully developed flow coefficients would reflect errors induced further upstream by the bare patch effect. In fact, of course, these coefficients agree closely with the Colburn equation values, indicating that the effect is small. By way of explanation, it has to be remembered that the effect is a transient phenomenon, and influences any particular position for a relatively short period of time. Furthermore, in most practical situations, it is unlikely that the distribution of heat-transfer coefficients will be as unfavourable as in the present case. It may be concluded, therefore, that the bare patch effect does not modify transfer coefficients significantly. Very recently, Stephenson *et al.* [25] have modelled the bare patch effect in the course of their analytical investigation of tube entrance region heat transfer. Their analysis indicates only a 1 per cent change in heat-transfer coefficients, thus verifying the conclusion from the present work.

CONCLUSIONS

The thin-film mass-transfer analogue technique has now been successfully developed to the extent that it is capable of predicting forced convection heat-transfer coefficients accurately and economically in a number of situations.

The accuracy of the method is dependent upon using a rigorous form of the heat/mass transfer analogy and upon having a refined technique for applying the naphthalene film to the surfaces of interest.

The effectiveness of the thin-film analogue technique has been demonstrated for developing flow in a circular tube. The analogue results are in very good agreement with other investigations for both the developing and fully developed regions.

The method is unique in that it gives 100 per cent coverage of the surfaces of interest. Furthermore, the transfer process is visible and could, therefore, be used to demonstrate various flow phenomena. It follows that the method could be used either as a powerful research tool or for instructional purposes.

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APPENDIX

Mass Transfer Analogue Calculation Procedure

Mass-transfer coefficient

$$h_m = \frac{W/At}{C_w - C_\infty}$$

where W = mass of naphthalene on plug; A = surface area of plug; t = corrected time for naphthalene to clear; C_w = wall concentration of naphthalene vapour (assumed saturated); C_∞ = free stream concentration (assumed zero).*

The wall concentration, C_w, is given by

$$C_w = \frac{p_w}{R_n T}$$

where p_w = saturated vapour pressure; R_n = gas constant for naphthalene (= 64·96 J/kg K); T = naphthalene temperature.

The saturated vapour pressure p_w is given by the formula of Sherwood and Bryant [27].

$$\log_{10} p'_w = -\frac{3765}{T} + 11{\cdot}55$$

where p'_w is in mm mercury.

The mass Stanton number

$$St_m = \frac{h_m}{u}$$

where u = bulk mean air velocity. Thus Stanton number

$$St\left(=\frac{h}{\rho uc}\right) = \phi\, St_m,$$

hence $h = \phi \rho C_p h_m$ where ρ = air density; c = specific heat capacity of air at constant pressure; and ϕ = heat/mass-transfer analogy factor (defined in the text).

The values for Schmidt number Sc, used in determining ϕ are given by the formula of Sherwood and Trass [28].

$$Sc = 7{\cdot}0 \times T^{-0{\cdot}185}.$$

*In the current tests, the bulk mean concentration of naphthalene after fourteen diameters was approximately 2 per cent of the wall concentration, diminishing towards zero as the clearance advanced along the tube. Strictly speaking, this should be accounted for in the definition of mass-transfer coefficient, but has been neglected in the current tests as being within the limits of experimental scatter.